

To theory of asymptotically stable accelerating Universe in Riemann-Cartan spacetime

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Abstract. Homogeneous isotropic cosmological models built in the framework of the Poincaré gauge theory of gravity based on general expression of gravitational Lagrangian with indefinite parameters are analyzed. Special points of cosmological solutions for flat cosmological models at asymptotics and conditions of their stability in dependence of indefinite parameters are found. Procedure of numerical integration of the system of gravitational equations at asymptotics is considered. Numerical solution for accelerating Universe without dark energy is obtained.

Keywords: Riemann-Cartan spacetime, isotropic cosmology, dark energy, torsion

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1 Introduction

One of the most principal achievements of observational cosmology is the discovery of the acceleration of cosmological expansion at present epoch. In order to explain accelerating cosmological expansion in the framework of General Relativity Theory (GR), the notion of dark energy (or quintessence) as some hypothetical kind of gravitating matter with negative pressure was introduced. Then the explanation of cosmological acceleration in the frame of GR leads to conclusion that approximately 70% of energy in our Universe is related to dark energy.

In the frame of standard Λ CDM-model the dark energy is associated with cosmological constant Λ , which is related to the vacuum energy density of matter fields. In terms of quantum field theory the vacuum energy density diverges and can be eliminated by means of renormalization procedure. At the same time the value of cosmological constant Λ , which is introduced into gravitational equations of GR manually, is very small and close to average energy density in the Universe at present epoch.

Another situation takes place in the framework of gravitation theory in the Riemann-Cartan spacetime U_4 - Poincaré gauge theory of gravity (PGTG) (see [1] and Refs herein). At first it should be noted that the PGTG is natural and in certain sense necessary generalization of metric gravitation theory by applying the local gauge invariance principle to gravitational interaction, if the Lorentz group is included into the gauge group which corresponds to gravitational interaction [2–6]. In the frame of PGTG the effective cosmological constant appears in cosmological equations by virtue of the most complicated structure of physical spacetime, notably by spacetime torsion [7, 8]. As it was shown in [8], the physical spacetime in the vacuum (in absence of gravitating matter) in the frame of PGTG in general case has the structure of Riemann-Cartan continuum with de Sitter metrics, but not Minkowski spacetime. Corresponding results were obtained by analyzing isotropic cosmology built in the frame of PGTG based on general expression of gravitational Lagrangian \mathcal{L}_g including both a scalar curvature and invariants quadratic in the curvature and torsion tensors with indefinite parameters (see [1, 8] and Refs herein).¹ From the point of view of PGTG the effect of gravitational repulsion leading to accelerating cosmological expansion at present epoch has

¹Similar results were discussed later in [16–18] by using the gravitational Lagrangian simplified in comparison with [7, 8].

the vacuum origin and it is connected with the change of gravitational interaction provoked by spacetime torsion without any dark energy.

The principal change of gravitational interaction takes place also in the beginning of cosmological expansion, when the energy density ρ and pressure p have extremely high values: by virtue of existence of limiting energy density, close to which the gravitational interaction in the case of usual matter satisfying standard energy conditions is repulsive, isotropic cosmology is regular [9]. The regularity takes place not only with respect to energy density and metric characteristics (scale factor of Robertson-Walker metric, Hubble parameter with its time derivative), but also with respect to torsion and curvature functions [10]. It should be noted that indicated physical results were obtained by certain restrictions on indefinite parameters of gravitational Lagrangian \mathcal{L}_g (see below). Additional restrictions on indefinite parameters can be found by investigation presented below of cosmological models for accelerating Universe.

The present paper is devoted to analysis of homogeneous isotropic models (HIM) with two torsion functions with the purpose to obtain asymptotically stable solutions for accelerating Universe. At first in Section 2 the principal relations of isotropic cosmology built in the frame of PGTG and using in this paper are given.

2 Principal relations of isotropic cosmology in Riemann-Cartan spacetime

In the framework of PGTG the role of gravitational field variables play the tetrad $h^i{}_\mu$ and the Lorentz connection $A^{ik}{}_\mu$; corresponding field strengths are the torsion tensor $S^i{}_{\mu\nu}$ and the curvature tensor $F^{ik}{}_{\mu\nu}$ defined as

$$S^i{}_{\mu\nu} = \partial_{[\nu} h^i{}_{\mu]} - h_{k[\mu} A^{ik}{}_{\nu]},$$

$$F^{ik}{}_{\mu\nu} = 2\partial_{[\mu} A^{ik}{}_{\nu]} + 2A^{il}{}_{[\mu} A^k{}_{|l|\nu]},$$

where holonomic and anholonomic space-time coordinates are denoted by means of greek and latin indices respectively.

We will consider the PGTG based on gravitational Lagrangian given in the following general form

$$\begin{aligned} \mathcal{L}_g = f_0 F + F^{\alpha\beta\mu\nu} (f_1 F_{\alpha\beta\mu\nu} + f_2 F_{\alpha\mu\beta\nu} + f_3 F_{\mu\nu\alpha\beta}) + F^{\mu\nu} (f_4 F_{\mu\nu} + f_5 F_{\nu\mu}) \\ + f_6 F^2 + S^{\alpha\mu\nu} (a_1 S_{\alpha\mu\nu} + a_2 S_{\nu\mu\alpha}) + a_3 S^\alpha{}_{\mu\alpha} S^\beta{}_{\nu\beta}, \end{aligned} \quad (2.1)$$

where $F_{\mu\nu} = F^\alpha{}_{\mu\alpha\nu}$, $F = F^\mu{}_\mu$, f_i ($i = 1, 2, \dots, 6$), a_k ($k = 1, 2, 3$) are indefinite parameters, $f_0 = (16\pi G)^{-1}$, G is Newton's gravitational constant (the velocity of light in the vacuum is equal to 1). Gravitational equations of PGTG obtained from the action integral $I = \int (\mathcal{L}_g + \mathcal{L}_m) h d^4x$, where $h = \det(h^i{}_\mu)$ and \mathcal{L}_m is the Lagrangian of gravitating matter, contain the system of 16+24 equations corresponding to gravitational variables $h^i{}_\mu$ and $A^{ik}{}_\mu$. By using minimal coupling of gravitational field with matter the sources of gravitational field in PGTG are the energy-momentum and spin momentum tensors.

In the framework of PGTG the dynamics of any HIM is described by means of three functions of time t : the scale factor of Robertson-Walker metrics R and two torsion functions S_1 and S_2 determining the curvature tensor. The system of gravitational equations of PGTG for HIM in considered case is reduced to 4 equations, which allow to obtain the generalization of Friedmann cosmological equations and equations for torsion functions [1, 8]. In general case

these equations contain five indefinite parameters – two parameters connected with terms of \mathcal{L}_g quadratic in the torsion tensor ($a = 2a_1 + a_2 + 3a_3$, $b = a_2 - a_1$) and three combinations of parameters f_i :

$$\begin{aligned} f &= f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6, \\ q_1 &= f_2 - 2f_3 + f_4 + f_5 + 6f_6, \quad q_2 = 2f_1 - f_2. \end{aligned} \quad (2.2)$$

Indefinite parameters have to obey some restrictions under physical and mathematical reasons. In accordance with [6] gravitational equations of PGTG based on gravitational Lagrangian (1) satisfy the correspondence principle with GR and lead in linear approximation in metric and torsion functions to Einstein gravitational equations, if the following conditions are satisfied: $a = 0$, $4(f_1 + \frac{f_2}{2} + f_3) + f_4 + f_5 = 0$ and $\alpha T \ll 1$, where $\alpha = \frac{f}{3f_0^2}$ ($f > 0$) and T is the trace of canonical energy-momentum tensor [11, 12]. The first two conditions are necessary to exclude higher derivatives of metrics from gravitational equations and the third condition in the form of inequality is valid for usual gravitating systems, if the parameter α having inverse dimension of energy density corresponds to extremely high energy densities.² In the frame of isotropic cosmology the condition $a = 0$ was used previously in order to exclude higher derivatives of the scale factor R from cosmological equations.³ Then cosmological equations and equations for torsion functions contain four parameters: b and parameters (2.2), which appear in the following combinations: α , $\varepsilon = \frac{q_2}{f}$ and $\omega = \frac{2f - q_1 - q_2}{f}$. The investigation of physical and mathematical consequences of isotropic cosmology allows to obtain some restrictions on these parameters. If the value of α^{-1} corresponds to the scale of extremely high energy densities, the explanation of accelerating cosmological expansion at present epoch together with the effect of existence of limiting energy density lead to the following conditions [11, 12]: $|\varepsilon| \ll 1$, $0 < 1 - \frac{b}{f_0} \ll 1$, $0 < \omega < 4$.

For further analysis, we transform cosmological equations (Eqs (3.1)-(3.2) in Ref. [1]) to dimensionless form by introducing dimensionless units for all variables and parameter b entering these equations and denoted by means of tilde:

$$\begin{aligned} t &\rightarrow \tilde{t} = t/\sqrt{6f_0\alpha}, & R &\rightarrow \tilde{R} = R/\sqrt{6f_0\alpha}, \\ \rho &\rightarrow \tilde{\rho} = \alpha\rho, & p &\rightarrow \tilde{p} = \alpha p, \\ S_{1,2} &\rightarrow \tilde{S}_{1,2} = S_{1,2}\sqrt{6f_0\alpha}, & b &\rightarrow \tilde{b} = b/f_0, \\ H &\rightarrow \tilde{H} = H\sqrt{6f_0\alpha}, \end{aligned} \quad (2.3)$$

where dimensionless Hubble parameter \tilde{H} is defined by usual way $\tilde{H} = \tilde{R}^{-1} \frac{d\tilde{R}}{d\tilde{t}}$. As result cosmological equations take the following dimensionless form, where the differentiation with respect to dimensionless time \tilde{t} is denoted by means of the prime and the sign of \sim is omitted below:

$$\begin{aligned} \frac{k}{R^2} + (H - 2S_1)^2 &= \frac{1}{Z} \left[\rho + (Z - b) S_2^2 + \frac{1}{4} (\rho - 3p - 2bS_2^2)^2 \right] \\ &\quad - \frac{\varepsilon}{2Z} \left[(HS_2 + S_2')^2 + 4 \left(\frac{k}{R^2} - S_2^2 \right) S_2^2 \right], \end{aligned} \quad (2.4)$$

²In the case of HIM with the only torsion function the value of α^{-1} determines the limiting energy density in the beginning of cosmological expansion at a bounce [13, 14].

³It should be noted that isotropic cosmology with $a \neq 0$ possesses some principal problems [15].

$$\begin{aligned}
H' + H^2 - 2HS_1 - 2S_1' &= -\frac{1}{2Z} \left[\rho + 3p - \frac{1}{2} (\rho - 3p - 2bS_2^2)^2 \right] \\
&\quad - \frac{\varepsilon}{Z} (\rho - 3p - 2bS_2^2) S_2^2 + \frac{\varepsilon}{2Z} \left[(HS_2 + S_2')^2 + 4 \left(\frac{k}{R^2} - S_2^2 \right) S_2^2 \right], \\
(Z \equiv 1 + \rho - 3p - 2(b + \varepsilon) S_2^2).
\end{aligned} \tag{2.5}$$

The torsion function S_1 (Eq. (3.3) in Ref. [1]) in dimensionless form in equations (2.4)–(2.5) is

$$S_1 = -\frac{3}{4Z} \left\{ H \left[(\rho + p) \left(3 \frac{dp}{d\rho} - 1 \right) + 2(\varepsilon + \frac{\omega}{3}) S_2^2 \right] - \frac{2}{3} (2b - (\varepsilon + \omega)) S_2 S_2' \right\} \tag{2.6}$$

and dimensionless torsion function S_2 (Eq. (3.4) in Ref. [1]) satisfies the following differential equation of the second order:

$$\begin{aligned}
\varepsilon [S_2'' + 3HS_2' + 3H'S_2 - 4(S_1' - 3HS_1 + 4S_1^2) S_2] - 2 \left(1 - \frac{\omega}{2} \right) (\rho - 3p - 2bS_2^2) S_2 \\
- 2(1 - b) S_2 - 2\omega \left[\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 \right] S_2 = 0.
\end{aligned} \tag{2.7}$$

The conservation law for gravitating matter in dimensionless units has the usual form

$$\rho' + 3H(\rho + p) = 0. \tag{2.8}$$

3 Critical points analysis

The system of equations (2.5) – (2.7) together with conservation law (2.8) completely determine the dynamics of HIM, if the equation of state of matter is given. The composition of gravitating matter and its equation of state change by cosmological evolution. By analysis of HIM at asymptotics we will consider further flat model ($k = 0$) filled with matter with barotropic equation of state $p = w\rho$ ($w = \text{const}$). The aforementioned system of equations can be represented in the form of four first order differential equations for H , S_2 , $U = S_2'$ and ρ :

$$M_0 \mathbf{Y}' = \mathbf{F}, \tag{3.1}$$

where the matrix M_0 is

$$M_0 = \begin{pmatrix} 1 - 2\frac{\partial S_1}{\partial H} & -2\frac{\partial S_1}{\partial S_2} & -2\frac{\partial S_1}{\partial U} & -2\frac{\partial S_1}{\partial \rho} \\ 0 & 1 & 0 & 0 \\ 3\varepsilon S_2 - 4\varepsilon \frac{\partial S_1}{\partial H} S_2 & -4\varepsilon \frac{\partial S_1}{\partial S_2} S_2 & \varepsilon(1 - 4\frac{\partial S_1}{\partial U} S_2) & -4\varepsilon \frac{\partial S_1}{\partial \rho} S_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.2}$$

and

$$\mathbf{Y} = \begin{pmatrix} H \\ S_2 \\ U \\ \rho \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \\ \mathcal{F}_3 \\ \mathcal{F}_4 \end{pmatrix},$$

where \mathcal{F}_i ($i = 1, 2, 3, 4$) are the following functions of H, S_2, U, ρ :

$$\mathcal{F}_1 = -H^2 + 2HS_1 - \frac{1}{2Z} \left\{ (1 + 3w) \rho - \frac{1}{2} [(1 - 3w) \rho - 2bS_2^2]^2 \right\} - \frac{\varepsilon}{Z} [(1 - 3w) \rho - 2(b - 1) S_2^2] S_2^2 + \frac{\varepsilon}{2Z} (HS_2 + U)^2, \quad (3.3)$$

$$\mathcal{F}_2 = U, \quad (3.4)$$

$$\mathcal{F}_3 = -\varepsilon [3HU + 4(3H - 4S_1) S_1 S_2] + 2 \left(1 - \frac{\omega}{2} \right) [(1 - 3w) \rho - 2bS_2^2] S_2 + 2(1 - b) S_2 + 2\omega [(H - 2S_1)^2 - S_2^2] S_2, \quad (3.5)$$

$$\mathcal{F}_4 = -3(1 + w) \rho H \quad (3.6)$$

and the function S_1 takes the form as

$$S_1 = -\frac{3}{4Z} \left\{ H \left[(1 + w) (3w - 1) \rho + 2(\varepsilon + \frac{\omega}{3}) S_2^2 \right] - \frac{2}{3} (2b - (\varepsilon + \omega)) S_2 U \right\}, \quad (3.7)$$

$$(Z = 1 + \rho(1 - 3w) - 2(b + \varepsilon) S_2^2).$$

Critical points $P_i = P_i(H_c, S_{2c}, U_c, \rho_c)$ of the first order system of differential equations (3.1) can be obtained by setting H', S_2', S_1', ρ' to zero [19, 20], i.e. by solving the following system of equations:

$$\mathcal{F}_i(H, S_2, U, \rho) = 0 \quad (i = 1, \dots, 4). \quad (3.8)$$

From (3.4) follows that $U_c = 0$. In the case of considering flat model solutions of (3.8) have to satisfy (2.4) with $k = 0$.

Obviously, the point P_0 with vanishing values of H_c, S_{2c}, ρ_c satisfies (3.8). Appropriate solution with vanishing S_2 -function at asymptotics appears at specific choice of parameters and does not have physical interest [8]. Analogously to GR this point is the point of complicated equilibrium. To analyze the stability of other critical points $P(H_c, S_{2c}, 0, \rho_c)$ satisfying (3.8) it is necessary to build linearized form of the system (3.1). Near the critical point the variables can be written in the form $H = H_c + \Delta H$, $S_2 = S_{2c} + \Delta S_2$, $U = \Delta U$, $\rho = \rho_c + \Delta \rho$ and the linearization of the system (3.1) takes the following relation

$$\Delta \mathbf{Y}' = M_0^{-1} M_1 \Delta \mathbf{Y}, \quad (3.9)$$

where the matrix M_0^{-1} is taken on the point P and the components of the matrix M_1 are given by

$$M_{1,ij} = \left(\frac{\partial \mathcal{F}_i}{\partial Y_j} \right) \Big|_P.$$

Stability of the point P is determined by the eigenvalues λ_i of the matrix $M_0^{-1} M_1$ [19, 20]. Characteristic equation $\det(M_1 - \lambda M_0) = 0$ leads to quartic expression with respect to λ , which can be written as

$$\lambda^4 + c_1 \lambda^3 + c_2 \lambda^2 + c_3 \lambda + c_4 = 0, \quad (3.10)$$

where c_i ($i = 1, 2, 3, 4$) are some functions of indefinite parameters. If the real parts of all λ_i are negative, then the critical point P is stable and the gravitational equations (2.5) – (2.8) have solution with asymptotics to this point $H \rightarrow H_c$, $S_2 \rightarrow S_{2c}$, $S_2' \rightarrow 0$, $\rho \rightarrow \rho_c$ at $t \rightarrow +\infty$.

According to the Routh-Hurwitz theorem all λ_i will have negative real parts if the main minors of the matrix

$$\begin{pmatrix} c_1 & 1 & 0 & 0 \\ c_3 & c_2 & c_1 & 1 \\ 0 & c_4 & c_3 & c_2 \\ 0 & 0 & 0 & c_4 \end{pmatrix} \quad (3.11)$$

are positive [19], i.e.

$$c_1 > 0, \quad c_1 c_2 - c_3 > 0, \quad c_1 c_2 c_3 - c_1^2 c_4 - c_3^2 > 0 \quad \text{and} \quad c_4 > 0. \quad (3.12)$$

The equation $\mathcal{F}_4(H_c, S_{2c}, 0, \rho_c) = 0$ is satisfied if at least one of possibilities is fulfilled: $H_c = 0$ or $\rho_c = 0$. Simultaneous fulfillment of these conditions leads to trivial solutions with $S_{2c} = 0$. Because the energy density in the case of considering flat model tends at asymptotics to zero, the physical interest assume critical points with non-vanishing Hubble parameter and vanishing energy density.

If $\rho_c = 0$ the system (3.8) is reduced to the system of two algebraic equations

$$-H^2 + 2HS_1 + [b^2 + 2\varepsilon(b-1)] \frac{S_2^4}{Z} + \frac{\varepsilon}{2Z} H^2 S_2^2 = 0, \quad (3.13)$$

$$\left[2\varepsilon(3HS_1 - 4S_1^2) + 2b(1 - \frac{\omega}{2})S_2^2 - \omega((H - 2S_1)^2 - S_2^2) - (1 - b) \right] S_2 = 0, \quad (3.14)$$

where the functions S_1 and Z can be represented in the following form

$$S_1 = -\frac{3}{2Z}(\varepsilon + \frac{\omega}{3})HS_2^2 \quad \text{and} \quad Z = 1 - 2(b + \varepsilon)S_2^2. \quad (3.15)$$

Neglecting the case $S_2 = 0$ and by using (3.15) the system of equations (3.13)–(3.14) can be rewritten in the following form

$$H^2 = \frac{b^2 - 2\varepsilon(1-b)}{1 + (\frac{1}{2}\varepsilon + \omega - 2b)S_2^2} S_2^4 \quad \left(1 + \left(\frac{1}{2}\varepsilon + \omega - 2b \right) S_2^2 \neq 0 \right), \quad (3.16)$$

$$\begin{aligned} & 9\varepsilon(\varepsilon + \frac{\omega}{3}) \left[1 - 2(b - \frac{\omega}{3})S_2^2 \right] H^2 S_2^2 + \omega H^2 [1 + (\varepsilon - 2b + \omega)S_2^2]^2 \\ & - [1 - 2(b + \varepsilon)S_2^2]^2 [S_2^2(\omega(1-b) + 2b) - (1-b)] = 0. \end{aligned} \quad (3.17)$$

Then Eqs.(3.16) – (3.17) allow to obtain the equation for S_2 in closed form:

$$\begin{aligned} & [1 - (2b + \omega + 4\varepsilon)S_2^2] \\ & \times \{ 2[2b^3(\omega - 4) + b^2(-\omega^2 + 4\omega(\varepsilon + 1) + \varepsilon(9\varepsilon + 2)) \\ & - 2b(\omega + 2)\varepsilon(\omega + 2\varepsilon) + 2\omega\varepsilon(\omega + 2\varepsilon)] S_2^6 + (-8b^3 + 2b^2(\omega + \varepsilon + 12) \\ & - 4b(\omega(\varepsilon + 2) + 2\varepsilon^2 + \varepsilon) + 4\varepsilon(\omega + 2\varepsilon)) S_2^4 \\ & + (8b^2 - b(2\omega + \varepsilon + 12) + 2\omega + \varepsilon) S_2^2 - 2(b - 1) \} = 0, \end{aligned} \quad (3.18)$$

and also the equation for H in closed form

$$\begin{aligned}
& \varepsilon(\omega + 2\varepsilon) [2b^3(\omega - 4) + b^2(-\omega^2 + 4\omega(\varepsilon + 1) + \varepsilon(9\varepsilon + 2)) \\
& - 2b(\omega + 2)\varepsilon(\omega + 2\varepsilon) + 2\omega\varepsilon(\omega + 2\varepsilon)] H^6 + [-2b^4(\omega^2 + 2\omega(\varepsilon - 2) + 2(\varepsilon - 2)\varepsilon) \\
& + b^3(2\omega^3 + \omega^2(\varepsilon - 8) - 2\omega\varepsilon(5\varepsilon + 9) - \varepsilon^2(17\varepsilon + 22)) \\
& + b^2(\omega^3(4\varepsilon - 2) + \omega^2(\varepsilon(18\varepsilon - 1) + 8) + 2\omega\varepsilon(\varepsilon(18\varepsilon + 11) + 12) + \varepsilon^2(\varepsilon(32\varepsilon + 41) + 34)) \\
& - 4b\varepsilon(\omega + 2\varepsilon)(2\omega^2 + 5\omega\varepsilon + \varepsilon(8\varepsilon + 3)) + 2\varepsilon(\omega + 2\varepsilon)(2\omega^2 + 5\omega\varepsilon + 8\varepsilon^2)] H^4 \\
& - 2(b^2 + 2(b - 1)\varepsilon) [4b(b^2 - b\omega + \omega) + (b - 1)\varepsilon(b(b - 4\omega - 2) + 4\omega) - 8(b - 1)^2\varepsilon^2] H^2 \\
& + 2(b - 1)^2(b^2 + 2(b - 1)\varepsilon)^2 = 0.
\end{aligned} \tag{3.19}$$

3.1 Approximate analysis in the case $0 < 1 - b \ll 1$

Analytic analysis of stable points determined by the system (3.16)–(3.17) is possible approximately only if $1 - b \rightarrow +0$ and $\varepsilon \rightarrow 0$. In other cases it is necessary to use numerical methods. By supposing that values of dimensionless functions S_2 and H at asymptotics in (3.16)–(3.17) are small ($|S_2| \ll 1$, $|H| \ll 1$) it is easy to obtain the following approximate solution of equations (3.16) – (3.17) if $0 < 1 - b \ll 1$ and $|\varepsilon| \ll 1$:

$$H_c = \frac{1 - b}{2\sqrt{b}}, \quad S_{2c} = \sqrt{\frac{1 - b}{2b}}. \tag{3.20}$$

This solution was obtained initially in ref. [7]. The stability of the critical point $P_1 \approx \left(\frac{1-b}{2\sqrt{b}}, \sqrt{\frac{1-b}{2b}}, 0, 0\right)$ can be analyzed analytically. The matrix M_0 and M_1 in this case according to their definition have the following form:

$$\begin{aligned}
M_0 &= \begin{pmatrix} 1 + \frac{1}{2}(1 - b)(\omega + 3\varepsilon) & 0 & \sqrt{\frac{1-b}{2}}(\omega + \varepsilon - 2) & -\frac{3}{4}(1 + w)(1 - 3w)(1 - b) \\ 0 & 1 & 0 & 0 \\ 3\sqrt{\frac{1-b}{2}}\varepsilon & 0 & \varepsilon + (1 - b)(\varepsilon^2 + \omega\varepsilon - 2\varepsilon) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \\
M_1 &= \begin{pmatrix} -(1 - b) & 0 & 0 & -\frac{3}{2}w - (1 - b)(\varepsilon + 1) - \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & -4(1 - b) & -\frac{3}{2}(1 - b)\varepsilon & -\sqrt{\frac{1-b}{2}}(\omega - 2)(1 - 3w) \\ 0 & 0 & 0 & -\frac{3}{2}(1 - b)(w + 1) \end{pmatrix}
\end{aligned}$$

As result we obtain the characteristic polynomial (3.10) in the form

$$\varepsilon \left[\lambda + \frac{3}{2}(1 + w)(1 - b) \right] \left[\lambda^3 + \frac{5}{2}(1 - b)\lambda^2 + 4\frac{1 - b}{\varepsilon}\lambda + 4\frac{(1 - b)^2}{\varepsilon} \right] = 0, \tag{3.21}$$

where higher order terms in powers of $(1 - b)$ are omitted. Due to factorization of this equation the analysis of real parts of λ is reduced to the analysis of cubic equation $\lambda^3 + c_1\lambda^2 + c_2\lambda + c_3 = 0$. The Routh-Hurwitz theorem in this case requires: $c_1 > 0$, $c_1c_2 - c_3 > 0$ and $c_3 > 0$. As result we obtain:

$$\varepsilon > 0, \quad w > -1. \tag{3.22}$$

3.2 Numerical analysis of stability

As an exact analytic expression for solution of the system (3.16)–(3.17) does not exist in general case, it is necessary to use numerical methods to analyze stability of the critical points. The procedure of the numerical analysis of the stability is following.

1. For given values of ε and ω , the system (3.16)–(3.17) is solved numerically for the set of values b .
2. For every real solution of the system (3.16)–(3.17) at given values of ε , ω and b characteristic equation $\det(M_1 - \lambda M_0) = 0$ has to be solved with respect to λ .
3. The real parts of obtained λ_i have to be tested for negativity.

For example, the results of this procedure for $\varepsilon = 0.001$ and $\omega = 2.5$ are given in figure 1. The calculation are performed for b varying from 0.01 to 1.2 with a step $\Delta b = 0.05$. In the right panel of figure 1 the curves for S_2 determined by (3.18) are imposed. In the left panel of this figure the curves for H determined by (3.19) are imposed.

From figure 1 one can see, that there is minimal value of b assuming nontrivial solution of eqs. (3.16)–(3.17). As numerical analysis shows, this value depends on parameter ε and weakly depends on ω . It follows from (3.16) that for sufficiently small ε and positive values of ω we have $b^2 - 2\varepsilon(1 - b) > 0$. As result we obtain the following restriction on b

$$b > -\varepsilon + \sqrt{\varepsilon(2 + \varepsilon)}. \quad (3.23)$$

Among various cosmological solutions of PGTG with stable asymptotics there are solutions which can correspond to observable Universe at present epoch. Such solutions we obtain by using the following restrictions on indefinite parameters: $0 < \varepsilon \ll 1$, $0 < 1 - b \ll 1$ and the parameter ω has to satisfy the condition $0 < \omega < 4$ [11, 12]. It should be noted that there are two different solutions at such restrictions on parameters, and only one of these solutions with small values of S_2^2 and H is physically acceptable [9].

4 Late-time approximation of cosmological solution

Now we will analyze the late-time behaviour of the solution of the system (2.5)–(2.8). To make comparison with Λ CDM-model of GR we will perform numerical integration of the system of the gravitational equations for dust matter ($w = 0$). To simulate late-time behaviour the initial conditions will be taken at the point $t_0 = 0$, which belongs to epoch of accelerating cosmological expansion. The total procedure includes the following steps.

1. For given acceptable values of ε , ω and b algebraic system (3.16)–(3.17) is solved numerically and all critical points $P_i(H_c, S_{2c}, 0, 0)$ are found. Only real solutions are considered.
2. For every critical point, the stability analysis is carried out according to the previous subsection and stable point with minimal positive H_c and positive (or negative) S_{2c} is selected.⁴

⁴These values of H_c and S_{2c} correspond to the vacuum as de Sitter spacetime with torsion [8, 9].

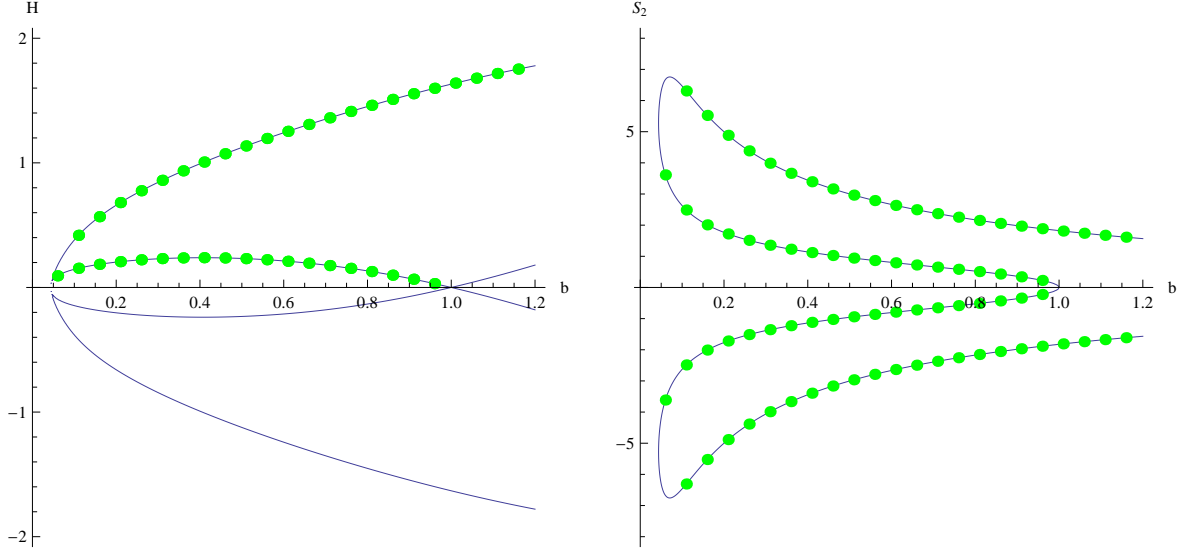


Figure 1. H_c and S_{2c} as function of b ($\varepsilon = 0.001$, $\omega = 2.5$). Stable critical points are marked by (green) circle. Unstable critical points and points of complicated equilibrium are not shown in this figure. Solid lines in the left and right panel of figure correspond to solutions of eqs. (3.18) and (3.19).

3. The torsion function S_2 and the Hubble parameter H at late-time approximation can be represented in the form

$$H^2 = H_c^2 + y_1 \rho, \quad (4.1)$$

$$S_2^2 = S_{2c}^2 + y_2 \rho, \quad (4.2)$$

with some coefficients y_1 and y_2 . As the stable point is selected, then ρ tends to zero at $t \rightarrow +\infty$. Keeping linear terms in ρ the conservation law (2.8) can be written as

$$\rho' = -3H_c \rho. \quad (4.3)$$

Substitution of (4.1)–(4.3) into (2.5)–(2.7) together with keeping terms linear in ρ gives two algebraic equations for determination of y_1 and y_2 . Numerical solution of these algebraic equations for given ε , ω , b , H_c and S_{2c} gives y_1 and y_2 .

4. Positivity of obtained values of y_1 and y_2 is considered as applicability of the late-time approximation (4.1)–(4.2) and successful choice of stable critical point made in step 2 of current procedure. Further steps are performed only if $y_1 > 0$ and $y_2 > 0$.
5. Initial condition for $\rho_0 = \rho(t_0)$ is taken from the following equation

$$\frac{H^2(t_0)}{H_c^2} \equiv \frac{H_c^2 + y_1 \rho_0}{H_c^2} = \frac{1}{\Omega_\Lambda}, \quad (4.4)$$

as result we have $H_0 = H(t_0) = \sqrt{H_c^2 + y_1 \rho_0}$ and $S_{20} = S_2(t_0) = \sqrt{S_{2c}^2 + y_2 \rho_0}$. Here Ω_Λ is an additional free parameter that specifies initial conditions.

6. Initial condition for $S'_{20} = S'_2(t_0)$ is obtained from (2.4) with $k = 0$. The minimal in modulus value of S'_{20} is taken as the initial value.

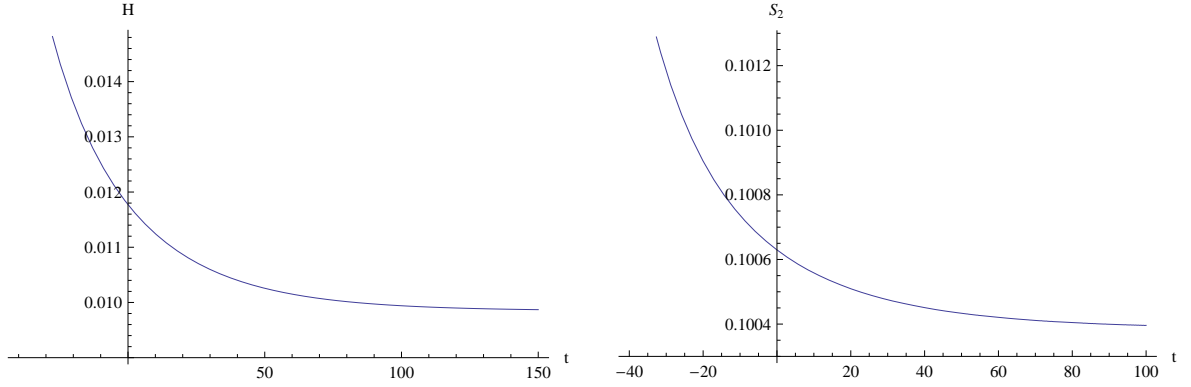


Figure 2. Late-time behaviour of Hubble parameter and S_2 torsion function.

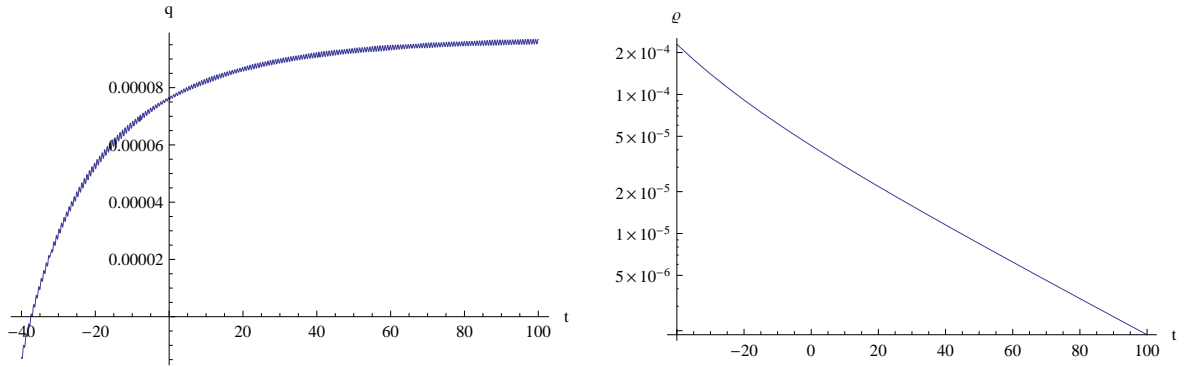


Figure 3. Late-time behaviour of the acceleration parameter and energy-density.

7. For this choice of the parameters ε , ω , b and initial conditions ρ_0 , H_0 , S_{20} and S'_{20} the system of differential equations (2.5)–(2.8) is integrated numerically.

As an example let us consider the numerical solution at the following parameters and initial conditions: $\varepsilon = 0.001$, $\omega = 2.5$, $b = 0.98$, $H_0 = 0.0118$, $S_{20} = 0.1006$, $S'_{20} = -9.5 \times 10^{-6}$, $\rho_0 = 0.000043$. This choice of the initial conditions gives $H^2(t_0)/H^2(\infty) = 1/\Omega_\Lambda = 1/0.7$. Figures 2–3 show the characteristic behaviour of Hubble parameter H , torsion function S_2 , acceleration parameter $q = R''R/R'^2$ and energy density of dust matter ρ for late-time phase of flat cosmological model. As one can see from Figure 3 for acceleration parameter, there was in the past the moment when $q = 0$ and the transition from deceleration to acceleration of cosmological expansion took place.

Obtained numerical solution for the Hubble parameter and energy density is close to that of standard Λ CDM-model. Certain distinction appears in the behaviour of acceleration parameter q because of its small oscillations which reduce by decreasing of parameter ε and disappear if $\varepsilon = 0$.

5 Conclusion

As follows from our analysis, isotropic cosmology built in the framework of the Poincaré gauge theory of gravity based on general expression of gravitational Lagrangian leads by

certain restrictions on indefinite parameters to asymptotically stable cosmological solutions for flat homogeneous isotropic models filled by dust matter, which can describe the stage of accelerated cosmological expansion of the Universe at present epoch without any dark energy. The spacetime in asymptotics in obtained solutions has the structure of Riemann-Cartan continuum with de Sitter metrics and non-vanishing torsion that demonstrates the dynamical role of the physical vacuum in the frame of PGTG .

Acknowledgments

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